On the Scales of Maps. By L. M. Iluupt, Prof. of Civil Engineering Toune Scientific Schonl.
(Read before the American Philusophical Society, Nov. 1, 1878.)
The object of this paper is to attempt if possible the removal of the amhiguities existing in regard to the use of ratios as expressing the scales of maps and degrees of slopes.

Mathematical authorities are by no means agreed concerning the definition of the term ratio. They all maintain that it is an expression for the relation existing between two quantities, but differ in the manner of determining the value of this relation ; souc, as Peck, Davies, Robinson and others, divide the second quantity or consequent by the first or antecedent ; some, as IIutton, Alsop, Ray and others, divide the first by the second quantity, and still a third class, as Chanvenct and others, define it as heing the quotient obtained by dividing one quantity by another. It inay therefore be either $\frac{a}{b}$ or $\frac{b}{2}, 2,000,000$, or $\frac{208}{2} \frac{2}{0} 000$.

The same confusion is found to exist in resignating the seales of maps and drawings. Some publishers and engineers giving it as so many miles, or other clenomination, to the ineh; nthers, as so many inches to the mile. Again in expressing slopes many anthorities use the tang. of the angle made with the horizon, that is the height divirlea by the base $\left(\frac{\Delta}{b}\right)$ while wthers use the co-tang, or $\frac{b}{a}$.

Now if we consider the manner of obtaining the value of the ratio in a (xeometrical Series or progression where no ambiguity exists, we find that as each subsequent term is obtained from its predecessor by multiplying by a constant factor called the ratio, so to obtain this factor or ratio we must necessarily divide any term hy the precoling one, and as this is the only way in which its value can be determinect, it establishes a rule which should be mate to apply to all other cases.

We should then define a ratio as being the expression for the ralue of the relation existing between tico quentities, and as obtained by dividing the second by the Finst.

The query then urises as to which quantity should be considered the first and which the second, and we answer that the given material object to be represented by the map or drawing is the Unit or measure with which the other is to be comparet. The map or drawing may be made of any convenient size, but the object to be represented is alteady fixed or constant in its dimensions, and hence, as the unit or standard of comparisou, should be made the divisor, or denominator of the quantity expressing the ratio : it is consequently the antecelent or first quantity. To illustrate, let it be required to determine the ratio hetween a map and its original in mature.

The trict to be delincated in miniature is the fixed object, invariable in size, which is to be compared with the plot representing it, ind which may be made larger or smaller according to circumstances, hence it become the unit of comparison, and is the antecedent or first quantity, and as such the denominator of the fraction expressing the ratio. The formula will then be:

Field: Plot $=\frac{\mathrm{P}}{\mathrm{F}} . \quad \mathrm{P}$ and F being always reduced to the same deno mination.

Thus a scale of $5^{\frac{1}{2} 80}$ is 5280 ft . of field to $1^{\prime}$ of map or one mile to 1 ft . $=\frac{1}{1^{2}}$ of a mile to $1^{\prime \prime}$, and not $1^{\prime \prime \prime}$ to 1 mile

It is evidently incorrect therefore to indicate the scales of maps as so many inches to a mile as is frequently done. Take the case of the recent Geological maps of one of our sister states said to be plotted on a seale of $3^{\prime \prime}$ to 1 m or $3^{\prime \prime}$ to $63,360^{\prime \prime}=\frac{6336 n}{3}=21,120$ that is to say the map is 21,120 times larger than the state itself, a manifest absurdity resulting from considering the map as the first quantity or standard rather than the fied itself.

In such cases crrors of interpretation can scarcely arise as the intention is so evident, but there nre numerous others that may lead to misconstruction, as where the drawings of small objects are nearly of the same size as the things represented-thus a scale of $\frac{t_{2}^{\prime \prime}}{}$ to $1^{\prime \prime}$ would confuse a mechanic unless he happened to know which was the larger, the object or the drawing.

So the expression $\frac{1}{4}^{\prime \prime}$ to $1^{\prime}$ is likewise incorrect as it is the reciprocal of the ratio intended-the inches evidently referring to the drawing and the foot to the object. As it stands, applying the definition of ratio as deduced, it will be equal to $12 \div-\frac{1}{4}=48$, making the dratwing 48 times the size of the model-it should be $1^{\prime}$ to $4_{1}^{\prime \prime \prime}$.

If it be remembered that the antecedent alrays refers to the ginen object and the consequent to the drusing, no difficulty ean arise. It will always happen then that if the drawing is on a smaller seale than the thing delineated, the ratio will be a proper fraction ; if larger, an improper fraction, and if equall the value will be unity, or $\frac{1}{1}$.

It is hardly neeessary to call attention to the fate that the mamber of scales in use is practically intinite, nud that serious inconvenience results therefrom to Engineers and Surveyors whose work extends over several counties or states, making it frequently necessary to redraw large sections of country. In compiling athases it is the practice of publishers to vary the scales necording to the amome of territory to be represented that the sheet may be filled up, bit mothing is ganed thereby since the scale used for the greutest area to be represented will show with equal clearness all the features of any other area. Moreover the eye becomes necustomed to estimating distances on the mals, with sullecient aceuracy for a recomaissance, when the senle is unifiom, but when variable it leme to great confusion, and especially when the publisher lins neglected to indicate the scate, as sometimes happens.

It is very desiruble to establish, if possible cither hy recommendations of scientifle societies or by general hass, some conventional seales for maps of varione mizes. 'Taking astate of medium mean N. Y. of Pemba, lor the unit, and redncing it to a conveniont size sheet of puper, saty $4 \times 3 \mathrm{ft}$., womble reguire a seate of conson, the same ns is used ly the U. S. Comsat Survey for general charts and reemmaismee, but too small for most other purposes. Larger ntates combld be plotel on the same soale by disserting
them．Foreign countries conducting Geodetic Surveys have adonted such a system．In Prussia，Austria and Switzerland the plane table sheet are plotted on a scale of $\overline{25 \hat{\delta} \delta \sigma}$ ．In Italy the ficld work is plotted on a scale

 $\overline{I o}_{\frac{1}{5} \delta \sigma}^{5}$ ，but these latter，while not being large enough to show parish hound． aries with sufficient accuracy，require about six times the amount of labor in their preparation and are inconvenient．The scale used by l＇russia and Switzerland for general maps is rootrod or one fourth that of the detail sheets obtained from the plane table surveys．

Populous，cultivated and mineral distriets in Great Britain are plotted on a scale of ${ }_{2}$ siod $^{\prime}=1 \mathrm{~m}$ ．to 2： $2.344^{\prime \prime}$ ，partially cultivated and thinly settled dis－ tricts，on a scale of $1^{\mathrm{mb}} \mathrm{to} 6^{\prime \prime}={ }^{50} \frac{1}{5} \mathrm{~s} 0$ ．For the plans of cities of over 4000 inhabitants a seale of sbo or 1 m. to 10.56 feet is ised，and for towns and villages ${ }^{1}{ }^{1} 5 \sigma$ or $1^{\mathrm{m}}$ ．to 5 ft ．is general．

Numerous other instances might be cited showing the great variety of scales in use，but these will suflice．It is evident that in Government or State Surveys some systematic connection may readily be established be－ tween the several seales used，and it is very desirable that this uniformity of scale be made more general．The scale adopted should be just larqe enough to show clearly all necessary detail．Anything more than this is a wasteful expenditure of time and money．

For general maps of States showing intercommunications，a scale of इनी⿱丷天心 will he found sufficiently large．

For maps of counties，in toto，a scale of zof $\frac{1}{b}$ will enable all necessary features to be elearly represented ；this scale applied to Lycoming Co ．，the largest in Penna．．would require a map $\left.6 \frac{1}{2} \times 4\right\} \mathrm{tt}$ ．For townships the sate of ${ }_{250 \frac{1}{0} \sigma}$ is quite large enough，and furnishe＇s an admirable size for the projection of Geological data．

For eities，towns and villages some decimal，sub－mutiples of the above seales sloould be used．

Cadastral maps of farms，parks or estates may be plotted on seales of ${ }_{2} \frac{1}{5}, 5 \frac{1}{6} 0,10^{\prime}$ ，

In indienting the dengees of slopes or the bater of retaining walls，the natural tangent of the angle which the slope makes with the horizon should invariably be used．

To save lime in determining the relative values of some of the most ime portant scales in use，and to aid in introducing the metrie system of lengths，I have with the assistance of Messrs．Wm．M．Pottsand J．W．Van Osten，Jr．， prepared the accompanying tables of equivalents．The first，gives the number of Miles，Kilometers，Poles，Chains，Yurds，Meters and Feet of ${ }^{\circ}$ territory which are equivalent to one inch of map for any given scale． The second，is the reciprocal of the first，and states the amount of map sur－ fice which would be covered by any one or more of the above units，for any scale．

Table of Map Equicalents giving for each

| No. | Srale. | Miles, | Kilometers, | Chains, | Pules, |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | \%.379.7800 | 116. | 186.6821 | 9280.0000 | 37120.0000 |
| 2 | צ.090.850 | 3:3. | 53.1078 | 2640.000 | 10560.00 |
| 3 | T.26 \%,200 | 20. | 32.18663 | 1600.000 | 6400.00 |
| 4 | 1.200.000 | 18.9393 | 30.4791 | 1515.15 | 6060.60 |
| 5) | $1.01 \frac{1}{3.760}$ | 16. | 25.7492 | 1280.100 | 5120.00 |
| 6 | $1.00 \frac{1}{1.000}$ | 15.7828 | 25.3992 | 1261.62 | 5046.50 |
| 7 | 811.008 | 12.8000 | 20.5994 | 1024.00 | 4096.00 |
| 8 | \% 1.320 | 12. | 19.3129 | 960.00 | 3840.00 |
| 9 | б35.000 | 10.0221 | 16.1286 | 801.768 | 3207.07 |
| 10 | \%33.800 | 10. | 16.09329 | 800.00 | 3200.00 |
| 11 | 600.000 | 9.4696 | 15.2398 | 757.575 | 3030.30 |
| 12 | 306.830 | 8. | 12.874.6 | 640.00 | 2560.00 |
| 13 | 500.000 | 7. 8914 | 12.6996 | 631.313 | 2595.25 |
| 14 | \%00.000 | 6.3131 | 10.1597 | 505.050 | 2020.20 |
| 15 | उ80.180 | 6. | 9.65587 | 480.00 | 1920.00 |
| 16 | 37\% $\frac{1}{3.000}$ | 5.918.5 | 9.5239 | 473.48 | 1893.92 |
| 17 | 316.800 | 5. | 8.04664 | 400.00 | 1600.00 |
| 18 | 300.000 | 4.7348 | 7.61992 | 378.78 | 1515.15 |
| 19 | $510.80 \pi$ | 3.78\%8 | 6.09570 | 303.03 | 1212.12 |
| 20 | $200.000$ | 3.15656 | 5.07985 | 250.525 | 1010.10 |
| 21 |  | 3. | 4.827935 | 240.00 | 960.00 |
| 22 | 180.000 | 2.5252 | 4.0638 | 202.02 | 808.08 |
| 23 | 150.000 | 2.36742 | 3.80496 | 189.39 | 757.57 |
| 24 |  | 2. | 3.21866 | 160.00 | 640.0 |
| 25 | $120.0 \delta 0$ | 1. 89393 | 3.05784 | 151.515 | 606.06 |
| 26 | 100.000 | 1.57828 | 2.53995 | 126.26 | $505.05$ |
| 27 | в0.000 | 1.2620 | 2.0319 | 101.01 | $404.04$ |
| $\stackrel{38}{98}$ | $\begin{aligned} & 80,200 \\ & 70.200 \end{aligned}$ | 1.2500 | 2.01166 | 100.00 | 400.00 |
| $\stackrel{29}{ }$ | 76.500 | 1.21212 | 1.9604 | 96.967 | .387.87 |
| 30 | 63.560 | 1. | 1. 5093 | 80.00 | 320.00 |
| 31 | 80.00\% | 0.94696 | 1.52392 | \% \% 7.75 | 303.03 |
| 32 | 5.500 | $0.93 \% \overline{3}$ | 1.508737 | \% 7.00 | 300.0 |
| 33 | 50.000 | 0.78914 | 1.26996 | 63.131 | 252.52 |
| 34 | 50.600 | 0.63181 | 1.0159 | 50.50 | 202.02 |
| 8.5 | 50.600 | 0.6050 | 1.00.i8 | 50.0 | 200.0 |
| 36 |  | $0.621: 38$ | 1. | 49.7104 | 198.88 |
| 18 98 | 3n:100 | 0.6060 | 0.9753 | 48.48 .4 | 198.933 |
| :38 | 38.018 | 0.6000 | 0.96 (.) 6 | $4 \% .925$ | 191.7) |
| :39 | 33.7 ${ }^{3}$ | 0.533.3 | 0.86146 | 42.666 | 1\%).66 |
| 40) | 30.000 | 0.47318 | 0.7619 | 37.8787 | 151.48 |
| 41 | 23.345 | 0.4000 | 0.6.1373 | 32.000 |  |
| 42 | 25,500 | $0.36 \cdot 4.57$ | 0. $3: 3917 \%$ | \$31.5656 | 120.209 |
| 43 | 75, 180 | 0.18500 | $0.680: 3 \cdot 19$ | 30. | 120.000 |
| 14 | 21.120 | (1,3i,i333 | (0,533.48 | 26.666 | 106. 6368 |
| 45 | 20.1080 | 0.31565 | (1.50)798 | 25.2505 | 101.0101 |
| 18 | 16.600 | 0.318250 | 0.50290 | 25. | 100. |
| 47 | 18.1800 | 0.30383 | 0.18762 | 24.242 | 90.939 |

lineal inch of Map, the following number of

| No. | Metres, | Yards and Feet $\left\{\begin{array}{l}\text { of Actual } \\ \text { Distanee. }\end{array}\right.$ |  | Where I'sed. |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 186682.18 | 204160.00 | 612480.00 | Map of U. S. in atlas. |
| 2 | 53107.86 | 58080.00 | 174240.00 | Map of Pa. |
| 3 | 32186.635 | 35200.00 | 105600.00 | U. S. C. S. |
| 4 | 30479.7 | 33333.33 | 100000.0 | U. S. C. S. |
| 5 | 25749.27 | 28160.0 | 84480.0 | $\triangle$ India. |
| 6 | 25399.2 | 27755.77 | 83333.3 | U. S. C. S. |
| 7 | 20599.416 | 23528.00 | 67584.00 |  |
| 8 | 19312.95 | 21120.00 | (63360.00 | R. R. Va. |
| 9 | 16128.6 | 17638.89 | 52916.66 | U. S. C. S. |
| 10 | 16093.29 | 17600.00 | 52800.00 | U. S. Eng. |
| 11 | 152:39.8 | 16666.6 | 50000.0 | U. S. C. S. |
| 12 | 12874.65 | 14080.0 | 42240.0 | Eng. Ord. Sur. |
| 13 | 12699.6 | 13888.8 | 41666.6 | U. S. C. S. |
| 14 | 10159.7 | 11111.1 | 33333.3 | U. S. C. S. |
| 15 | 9655.87 | 10560.0 | 31680.0 | Ludlow's Kep. |
| 16 | 9523.9 | 10416.5 | 31250.0 | U. S. C. S. |
| 17 | 8046.64 | 8800.00 | 26400.0 | Barnes' Pa. Maps, 1851. |
| 18 | 7619.9 | 8344.3 | 25000.0 | U. S. C. S. |
| 19 | 6095.7 | $666 \mathrm{ti6.6}$ | 20000.0 | U. S. C. S. |
| 20 | 5079.8 | 5555.5 | 16666.6 | U. S. C. S. |
| 21 | 482\%.935 | 5280.0 | 15840.0 | Ludlow's Rep. |
| 22 | 4063.8 | 4444.4 | 13333.3 | U. S. C. S. |
| 23 | 3804.9 | 4166.6 | 12500.0 | U. S. C. S. |
| 24 | 3218.66 | 3520.0 | 10560.0 | Sherman's March. |
| 25 | 3057.8 | 3333.3 | 10000.0 | U. S. C. S. |
| 20 | 2539.9 | 2777.7 | 8333.3 | U. S. C. S. |
| 27 | 2031.9 | 2222.2 | 6666.6 | U. S. C. S. |
| 28 | 2011.7 | 2200.0 | 6600.0 |  |
| 29 | 1960.5 | 2133.33 | 6400.0 | Geol. Sur. |
| 30 | 1609.3 | 1760.0 | 5280.0 | Fremont. |
| 31 | 1523.9 | 1666.6 | $5000: 0$ | U. S. C. S. |
| 32 | 1508.73 | 1650.0 | 4950.0 |  |
| 33 | 1269.9 | 1388.8 | 4166.6 | U. S. C. S. |
| 34 | 1015.9 | 1111.1 | 3333.3 | U. S. C. S. |
| 35 | 1005.83 | 1100.0 | 8300.0 | U. S. C. S. |
| 36 | 1000.0 | 1093.6 | 3280.8 |  |
| 37 | 975.24 | 1066.66 | 3200.0 | Geol. Surv. |
| 38 | 965.59 | 1054.33 | 3163.0 |  |
| 39 | 861.458 | 938.66 | 2816.1 |  |
| 40 | 761.9 | 833.3 | 2500.0 | U. S. C. S. |
| 41 | 643.728 | 704.000 | 2112.000 |  |
| 42 | 639.673 | 694.44 | 2083.333 |  |
| 43 | (603.487 | 660.00 | 1980.000 |  |
| 44 | 535.8969 | 586.66 | 1760.000 |  |
| 45 | 507.98 | 555.5 | 1666.66 |  |
| 46 | 502.906 | 550.00 | 1650.00 | U. S. C. S. |
| 47 | $48 \% .61 \%$ | 533.333 | 1600.00 |  |

Table of Map Equioulents giviny for each

| No. | Scale. | Miles, | Kilometers, | Chalms, | Poles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 48 | 18.818 | 0.29700 | 0.47790 | 23.760 | 95.04 |
| 49 | 131850 | 0.25000 | 040232 | 20. | 80. |
| 50 | 15.000 | 0.23674 | 0.38099 | 18.9393 | 75.75 |
| 51 | 11.880 | 0.1850 | 0.30174 | 15. | 60. |
| 52 | 10.000 | 0.1578 | 0.25417 | 12.626 | 50.505 |
| 53 | 5.9500 | 0.15625 | 0.25100 | 12.500 | 50.000 |
| 54 | 9.8 | 0.15151 | 0.24376 | 12.121 | 48.484 |
| 55 | - | 0.12500 | 0.20112 | 10. | 40.000 |
| 56 | F. $\frac{1}{2}$ 200 | 0.1136 | 0.18378 | 9.0909 | 36.363 |
| 57 | 6.000 | 0.09471 | 0.15285 | 7.5757 | 30.303 |
| 58 | 5.180 | $0.093, \pi 5$ | 0.15092 | 7.5000 | 30.000 |
| 59 | 3.1800 | 0.078913 | 0.12695 | 6.31313 | 25.252 |
| 60 | ¢.1.850 | 0.078123 | 0.12.58\% | 6.250 | 25.000 |
| 61 |  | 0.07 .375 | 0.121881 | 6.0606 | 24.243 |
| 62 | \%. 9 \%0 | 0.062 .50 | 0.100561 |  | 20.000 |
| 63 | 3. | 0.05681 | 0.091391 | 4.5303 | 18.1212 |
| 64 | \%.is3 | 0.05261 | 0.08463 | 4.2060 | 16.8242 |
| 65 | उ.158 | 0.05 | 0.080466 | 4. | 16.000 |
| 66 | з. O \% | 0.04734 | 0.07610 | 3.7787 | 15.151 |
| 67 | 2.818 | 0.04687 | 0.07 .51 | 3.75 | 15000 |
| 68 | ¢, 3 ou | 0.03945 | 0.06396 | 3.1565 | 12.626 |
| 69 | 2. 5 | 0.03787 | 0.06098 | 3.0379 | 12.1515 |
| 70 | 1.980 | 0.03125 | 0.0.50:9 | 2.5 | 10.000 |
| 71 | T. 180 | 0.02020 | 0.032507 | 1.6016 | 6.406 |
| T2 | T, ${ }^{1} 50$ | 0.019728 | 0.031697 | 1.5767 | 6.307 |
| 73 | 1. 1.00 | 0.018933 | 0.030578 | 1.5151 | 6.060 |
| 74 | T. O \% 6 | 0.017046 | 0.027520 | 1.3636 | 5.454 |
| \%) | गकी | 0.01515 | 0.024376 | 1.2121 | 4.848 |
| 76 | ${ }_{8}^{1}+0$ | 0.013258 | 0.021399 | $1.0605 \%$ | 4.2420 |
| 77 | -12 | 0.0125 | 0.02011 | 1. |  |
| 78 | -10 | 0.01136 | ().018:378 | 0.9091 | 3.6363 |
| 79 | -1\% | $0.0094 \% 1$ | 0.015285 | $0.75 \% 5 \%$ | 3. 0:303 |
| 80 | sod | 0.0078913 | 0.012695 | 0.63131 | 2.5259 |
| 81 | rio | $00075 \%$ | 0.012188 | 0.60606 | 24242 |
| 89 | 3 ¢\% | 0.00568 | 0.009139 | 0.45303 | 1.81212 |
| $8: 3$ | 30 | 0.00473 .4 | 0.007610 | 10.37787 | 1.51515 |
| 84 | \% 10 | 0.003787 | 0.00)6098 | 0.30379 | 1.21.515 |
| 8.5 | $16 \%$ | 0.008125 | 0.00.50:9 | 0. 2.5 | 1. |
| 86 |  | 0.001804 | 0.003057 | 0.15151 | 0.6060 |
| 87 | \% ${ }^{1}$ | 0.0009 .47 | 0.001528 | (0.0\%5\% | 0.3030 |
| W8 | 90.3006 | 0.0006918 | 0.101 | 0.6.697101 | 0.1988105 |
| 89 | ${ }_{3}^{1 / 85}$ | (0.000.518 | 0.00099139 | 0.0 .158103 | 0.181212 |
| 40 | 15 | 0.0001894 | 0.000:30.5 7 | 0.015151 | 0.06606 |
| 11 |  | 0.00001578 | 0.00002 .3536 | 0.0012530. | 0.0050 .3 |
| 42 |  | $0.00601183 \%$ | 0.00010190 | 0.00099407 | 0.003787 |
| 4:3 |  | (0.0000)(0)8! | 0.000012088 | 0.00062! | 0.002525 |

lineal inch of Map, the following number of

| No. | Metres, | Yards and Feet $\left\{\begin{array}{l}\text { of Arcual } \\ \text { Distance. }\end{array}\right.$ |  | Where Ised. |
| :---: | :---: | :---: | :---: | :---: |
| 48 | 477.96 | 522.72 | 1568.1 | U. S. C. S. |
| 49 | 402.325 | 440.00 | 1320.00 |  |
| 50 | 380.99 | 416.66 | 1250.00 | U. S. C. S. |
| 51 | 301.744 | 330.00 | 990.00 |  |
| 52 | 254.177 | 277.77 | 833.33 | U. S. C. S. |
| 53 | 2.51 .004 | 275.000 | 825.00 |  |
| 54 | 243.763 | 266.66 | 800. |  |
| 55 | 201.125 | 220.00 | 660. |  |
| 56 | 183.782 | 200. | 600. |  |
| 57 | 152.854 | 166.66 | 500. |  |
| 58 | 150.924 | 165.00 | 495. |  |
| 59 | 126.950 | 138:888 | 416.66 | U. S. C. S. |
| 60 | 125.8238 | 134.166 | 412.50 |  |
| 61 | 121.88175 | 133.333 | 400. |  |
| 62 | 100.562 .5 | 110.0 | 330. |  |
| 63 | 91.391 | 100. | 300.00 |  |
| 14 | 84.6334 | 92.592 | 277.777 | U.S. C. S. |
| 65 | 80.0466 | 88. | 264. |  |
| 66 | 76.1057 | 83.333 | 250.00 |  |
| 67 | 75.4138 | 82.5 | 247.5 |  |
| 68 | 63.9673 | 69.444 | 208.33 | U. S. C. S. |
| 69 | 60.9811 | 66.666 | 200. | U. S. C. S. |
| 70 | 50.2906 | 55.55 | 166.66 |  |
| 71 | 32.5079 | $35.55 \%$ | 106.66 | U. S. C. S. |
| 72 | 31.6978 | 34.7222 | 104.166 | U. S. C. S. |
| 73 | 30.578 | 33.3333 | 100. |  |
| $\begin{aligned} & 74 \\ & 75 \end{aligned}$ | $\begin{aligned} & 27.520 \\ & 24.3763 \end{aligned}$ | $\begin{aligned} & 30 . \\ & 26.666 \end{aligned}$ | $90 .$ |  |
| 76 | 21.4046 | 23.3333 | 70. |  |
| 77 | 20.1125 | 22. | 66. |  |
| 78 | 18.3782 | 20. | 60. |  |
| 79 | 15.2854 | 16.666 | 50. |  |
| 80 | 12.695 | 13.8888 | 41.666 | U. S. C. S. |
| 81 | 12.18817 | 13.3333 | 40. |  |
| 82 | 9.1391 | $10 .$ | 30. |  |
| 83 | 7.61057 | 8.3333 | 25. |  |
| 84 | 6.09811 | 6.6666 | 20. |  |
| 85 | 5.02906 | 5.555 | 16.666 |  |
| 86 | 3.0578 | 3.3333 | 10. | U. S. C. S. |
| 87 | 1.52854 | 1.6666 |  |  |
| 88 |  | 1.093623 | 3.280869 |  |
| 89 | 0.91391 |  | 3. |  |
| 90 | 0.30578 | 0.3333 | 1. |  |
| 91 | 0.025368 | 0.02777 | 0.083 |  |
| 92 | 0.019026 | 0.020833 | 0.062i |  |
| 93 | 0.012684 | 0.013888 | 0.0415 |  |

A Reciprocal Table of Map Equivalents showing the number of inches of

| No． | scale． | 1 Mile． | 1 Kilometer． | 1 Chain． | 1 Pole． |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 7． 3 方：．：60 | 0.0086205 | 0.005359 | 0.00010775 | 0.00002693 |
| 2 | 2.090 .880 | ． 03030 | ． 01883 | ． 000378 | 0000945 |
| 3 | 1．26 $\frac{1}{6} \cdot 200$ | ． 05000 | ． 03106 | ．000625 | ． 00015625 |
| 4 | 5.20 1．000 | ． 052880 | ． 03280 | ． 000660 | ． 0001650 |
| 5 | 1．013：\％${ }^{1}$ | ．06250 | ． 03883 | ．000781 | ． 00019525 |
| 6 | r．00\％$\frac{1}{600 \%}$ | ． 06336 | ． 03937 | ．000792 | ． 00019800 |
| \％ | \％11．088 | ．078125 | ． 04854 | ． 0009765 | ． 0002441 |
| 8 | 760．320 | $.08333+$ | ． $0517 \%$ | ． 001041 | ．00026025 |
| 9 | \％33．000 | ． 09979 | ． 06199 | ． 001247 | ． 00031175 |
| 10 | 633.600 | ． 10000 | ．06213 | ． 001250 | ． 0003125 |
| 11 | бо0．000 | ．10560 | ． 06501 | ． 00132 | ． 0003300 |
| 12 | 53.8880 | ． 125000 | ． 07766 | ． 001562 | ． 0003905 |
| 13 | 万01\％ | ． 12672 | ． 07874 | ． 001584 | ． 0003960 |
| 14 | 70.000 | ． 15840 | ．09842 | ． 00198 | ． 06049.0 |
| 15 | $380.180$ | $.16666+$ | ． 10355 | ． 002083 | ．00052075 |
| 16 | ${ }_{575000}$ | ．16896 | ． 10498 | ．002112 | ．00052800 |
| 17 | ग58．800 | 20000 | ． 12426 | ． 00250 | ．0006250 |
| 18 | з00．00\％ | ． 21120 | ．1312： | ．00264 | ． 0006600 |
| 19 | 270．000 | ． 26400 | ． 16403 | ． 01133300 | ．008250 |
| 20 | 208.000 | ． 31680 | ． 19684 | ．003960 | ． 0009900 |
| 21 | 190．080 | ．3：3333＋ | ． 20711 | ．004166 | ．0010415 |
| 22 | rodiono | ． 39600 | ． 24605 | ． 004950 | ． 0012325 |
| 23 | 130：000 | ． 42040 | ．2624．5 | ．005280 | ．0013200 |
| 24 | 126．750 | ． 50000 | ． 31067 | ．006250 | ． 0015625 |
| 25 | 52\％．00 | ． 52800 | ． 39807 | ．0066300 | ． 0016500 |
| 26 |  | ． 633360 | ． 30368 | ．00792 | ． 0019800 |
| 27 | उ0．000 | ． 99200 | ． 49210 | ． 0099000 | ． 0024750 |
| 28 | 7.9 .200 | ． 8 | ． 497101 | ． 01 | ． $00 \div 5$ |
| 99 |  | ．82500 | ． 51261 | ．1）10312 | ．002．5780 |
| 30 |  | $1.000(k)$ | 62130 | ．012500 | ．0031250 |
| 31 | \％0． 1000 | 1.05600 | ． 05614 | ．）13300 | ．003300 |
| 3 | ¢5．700 | 1.066666 | ．662801 | ．（）13333 | ．00333：3 |
| 3：3 | $\therefore 10.080$ | 1．20720 | ． 88.337 | ． 01585 | ．0039095 |
| 34 | 50．000 | 1.58400 | ．98421 | ． 019800 | ．004950 |
| 3.7 | s0．600 | 1.6 | ．994202 | ． 0 | ． 00500 |
| 313 | 98.388 | 1．60934 | 1.00000 | ．0：0116 | ． 0050290 |
| 37 | 8x， 1800 | 1． 65000 | 1．025 20 | ． 020162 | ． 00515550 |
| 38 | $8 x^{3} .618$ | 1．966666 | 1.03509 | ．0208：338 | ．0052183 |
| 39 | $38^{3} \cdot 7$ ¢ 4 | 1.875000 | 1.16383 | ． $0: 23 \cdot 13137$ | ． 0058859 |
| 40 | 30.000 | 2.11200 | 1.31288 | ． 0226.400 | ． 00 （icior）0 |
| 41 |  | 2.510000 | 1．5ib334 | ． 031250 | ．007R125 |
| 42 | 25．100 | 2.58440 | 1.57474 | ． 031 （fis0 | ．0079200 |
| 43 | 28.980 |  | 1．05502 | ．033：338＋ | ． $60833383+$ |
| 4.4 | 21.120 | 3.00000 | 1.86403 | ．03750） 0 | ．00938750 |
| 4.5 | 30 品0\％ | 3.13800 | 1．9）（88．12 | 0：39）（i） | ．009900） |
| 43 | $10^{1}$ ¢0\％ |  | 1.98810 .4 | ． 0.4 | ． 010 |
| 47 | rolana | 3.300001 | 2.050 .4 | （0．4125 | ．010312． |

Map and parts thereof, of the rarious scales now in use, which represent

| No. | 1 Metre. | 1 Yard. | 1 Foot. | Where Used. |
| :---: | :---: | :---: | :---: | :---: |
| 1 | . 000005359 | 00000489 | . 00000163 | Military. |
| 2 | . 00001882 | . $200017 \%$ | . $00000573+$ | Sherman's March Map. |
| 3 | . 00003106 | . 0000284 | . $00000946+$ |  |
| 4 | . 00003280 | . 0000300 | . 00001000 | U. S. C. S. |
| 5 | . 00003883 | . 0000355 | $.00001183+$ | $\triangle$ India. |
| fi | . 00003937 | . 000083600 | . 00001200 | U.S.C.S. |
| 7 | . 00004854 | . 00004438 | . 00001446 |  |
| 8 | . 00005177 | . 00004 \% $\%$ | . $00001576+$ | R. R. Virginia. |
| 9 | . 00006199 | . 00005056 | . $000015.53+$ | U. S. C. S. |
| 10 | .00006213 | . 0000568 | . 000015600 | U. S. Eng's. |
| 11 | . 000006561 | .0000600 | .0000200 | U. S. C. S. |
| 12 | . 000007766 | . 0000710 | . $0000236+$ | Eng. Ord. Sur. |
| 13 | . 00007874 | . 0000720 | . 0000240 | I. S. C. S. |
| 14 | . 0 OC098842 | .0060900 | . 00000300 | U. S. C. S. |
| 15 | .00010355 | . 0000946 | $.00003153+$ | Ludlow's Rep. |
| 16 | . 00010498 | . 0000960 | .00008200 | U. S. C. S. |
| 17 | . 00012426 | . 0001136 | .00003753+ | Pames' P'a. Map, 1851. |
| 18 | . 00013122 | . 0001200 | . 0004000 | U. S. C. S. |
| 19 | . 00016403 | . 0001500 | .0000500 | U. S. C. s. |
| 20 | . 00019684 | . 000180 | .0000600) | U. S. C. S. |
| 21 | . 00020711 | . 0001893 | . 0006310 | Ludlow. |
| 22 | . 00024605 | .0002250 | . 00007300 | U. S. C. S. |
| 23 | . 00026245 | . 0002400 | . 0000800 | U. S. C. S. |
| 24 | . 00031067 | . 0002840 | 0000946+ | Sherman's March |
| 25 | .0003:807 | . 0003000 | . 0001000 | U. s. C. S. |
| 26 | . 00039368 | . 0003600 | . 0001200 | U. S. C. s. |
| 27 | . 00049210 | . 0004500 | .000150)00 | U. S. C. S. |
| 28 | . 0004971 | .0004545 | . 000015151 |  |
| 29 | . 00051261 | . 00046875 | . 00015625 | Geol. Surv. |
| 30 | . 00062130 | . 00056800 | . $00018933+$ | Fremont. |
| 31 | . 00065614 | . 000600 | . 000200 | U. S. C. S. |
| 82 | . $00066{ }^{2}$ | . 00060606 | . 000030202 |  |
| 33 | . 00078737 | .000720 | . 0002400 | .. .. |
| 34 | . 00098421 | . 000900 | . 0000300 | " ${ }^{\text {a }}$ |
| 35 | . 0009941 | . 0009090 | . 0003030 |  |
| 36 | . 0010000 | . 0009144 | .0003048 |  |
| 37 | .00102.522 | . 0009375 | . 000312.5 | Geol. |
| 38 | . 001035 | . 000947 | . 0003156 |  |
| 39 | . 00116 in 3 | .0010653 | .0003551 |  |
| 40 | . 00131228 | . 0012000 | . 0004000 | U. S. C.S. |
| 41 | .00155334 | . 0014190 | . 0004730 |  |
| 42 | . 00157474 | .00144000 | . 0004800 |  |
| 43 | .00165692 | . 00151515 | . 00050505 |  |
| 44 | . 00186403 | . 0017040 | .0005680 |  |
| 45 | . 00196842 | . 0018000 | . 0006060 |  |
| 46 | . 001988 | . 001818 | . 0006060 |  |
| 47 | .00205044 | . 00187.00 | . 0062500 |  |

A Reciprocal Table of Map Equivatents shoving the number of inches of

| No. | Scale. | 1 Mile. | 1 Kilometer. | 1 Chain. | 1 I'ole. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 48 | 18, $\frac{1}{8,518}$ | 3.36698 | 2.09206 | . 042087 | 01052\% 5 |
| 49 | 18.818 15.80 | 4.0 | 2.485 .507 | . 05 | . 0125 |
| 50 | $\frac{15000}{15.000}$ | 4.22400 | 2.624 .56 | 1.052800 | . 0132000 |
| 51 | 11, ${ }^{1} 80$ | 5.33333 | 8.314009 | . 06666 | . 016666 |
| 52 | 10.000 | 6.33600 | 3.93685 | . 079200 | . 0198000 |
| 53 | 2.900 | 6.4 | 3.976808 | . 08 | 020 |
| 54 | 9.500 | 6.60000 | 4.10088 | .082500 | . 020635 |
| 3.5 | - 720 | 8. | 4.971014 | . 10 | .025 |
| 56 | $\cdots$ | 8.80000 | 5.46784 | .11000 | . 027500 |
| 57 | \%.1000 | 10.56000 | 6.561423 | . 132000 | . 0833000 |
| 58 | 3. 170 | 10.6666 | 6.628018 | . 1333333 | . 03333 |
| 59 | 3.000 | 12.67200 | $7.873 \%$ | . 15840 | .039600 |
| 60 | $\text { T. } 950$ | 12.8 | 7.953616 | . 16 | . 04 |
| 61 | $7 . \frac{1}{500}$ | 13.20000 | 8.201770 | .165000 | . 041250 |
| 62 | उ: $\frac{1}{\text { ¢ }}$ ¢ 0 | 16. | 9.942028 | .2 | . 0 5 |
| $(63$ | 3.6 | 17.6 | 10.93568 | .22 | . 0.55 |
| 64 | 3.15 | 19.00990 | 11.81173 | . 237623 | . 05940575 |
| 65 | 3:188: | 20. | 12.4243 .4 | . 2 J | .0625 |
| 66 | 3.000 | 21.12 | 13.122846 | . 264 | . 066 |
| 67 | $2 . \frac{1}{9} \% 0$ | 21.333:33 | 13.256036 | .26666 | .06666 |
| 68 | 2. 1000 | 25.34400 | 15.74740 | . 311880 | . 079200 |
| 69 |  | 26.40000 | 16.40:354 | .3:3000 | .082500 |
| 70 | $1.950$ | 32. | 19.88105 | . 4 | . 1 |
| 71 | T. ${ }^{\frac{1}{2} 50}$ | 49.50000 | 22.94414 | .618750 | .1546875 |
| 72 | 1.250 | 50.68500 | 31.49480 | . 63360 | . 158400 |
| 13 | 1.1500 | 52.80000 | 32.80708 | . 6680000 | . 165000 |
| 74 | T. $\mathrm{S}_{\text {¢ }}$ | $58.66666+$ | 36.4 .5231 | $.733333+$ | $.18333+$ |
| \% 5 | वर्परण | 66.00000 | 41.00885 | . $8: 5000$ | 200250 |
| 76 | ${ }^{1} 10$ | 7.5.428.) | 41.813726 | . 942857 | -33571425 |
| $7 \%$ | -18 | 80.30418 | $49.896 \% 0$ | 1.003802 | .2509505 |
| 48 | - 20 | 88.100000 | $54.6784 \%$ | 1.100000 | 275000 |
| T0 | ado | 105.60000 | (65. 61416 | 1.320000 | . 33000 |
| 80 | 晾碞 | 126.72000) | 78. $73 \% 00$ | 1.5)8.4000 | . 39600 |
| 81 | $4 \times 8$ | 132.00000 | $82.1117 \%$ | 1.650000 | .412500 |
| 8. | 3 ho | 173.00000 | 109.355694 | 2.2000 | . 5.50000 |
| $8: 3$ | 3 Jo | 211.20000 | 131.22833 | 2.640000 | .6600) |
| 84 | 2)0 | 26.4 .00000 | 164.0) 3.5 .11 | $3.300000$ | .825000 |
| 8.5 | In | 3\%0. | 198.8405 | 4. | 1. |
| 817 | $1 \frac{1}{10}$ | 228.000000 | 328.0708: | fi. (i)00000 | 1. $1: 50000$ |
| xi | \% 0 | 105\%.00000 | 656) $1416 \mathrm{li}+$ | 13.200000 | 3.3000 |
| 88 | 80.1506 | 16) 5 ). 330 | 1006. | 20.11668 | 5. 12916 |
| $8!$ | +18 | 1760. | $108: 5.569 .4$ | 22. | 5.5 |
| (H) | $1^{18}$ | 5980.00000 | 3280.7083 | 60.10000 | 16.5000 |
| \%) |  | $63338(1.000000$ | 13133485000 | \%02.00000 | 188.0000 |
| (1) | , | 84480.06) | 50491.033 3 + | 1050.000) | 23.4 .1060 |
| :17\% | ; | 1207\%0).00 | 78737.0000 | 1581.000 | $36)(6.000$ |

Map and parts thereof, of the various scales now in use, which represent

| So. | 1 Netre. | 1 Yard. | 1 Foot | Where I'sed. |
| :---: | :---: | :---: | :---: | :---: |
| 48 | . 00209206 | . 00119130 | . $00006376+$ | U. S. C. S. |
| 49 | . 002485 | . $0022 \pi 2$ | . 0007575 |  |
| 50 | .00262456 | . 0024000 | . 0008000 | U. S. C. S. |
| 51 | . 003314 | .0030303 | . 0010101 |  |
| 52 | . 00393685 | . 0036000 | . 0012000 | U. S. C. S. |
| 53 | . 0038976 | . 00336336 | . 001212 |  |
| 54 | . 00410088 | . 00375 | . 0012500 |  |
| 55 | . 004970 | . 004544 | . 0015150 |  |
| 56 | . 00546784 | . 005000 | .001666+ |  |
| 57 | . 006561423 | . 0066000 | . 002000 |  |
| 58 | . 006628 | .0060606 | .0020202 |  |
| 59 | . 0078737 | .007200 | . 002400 | U. S. C. S. |
| 60 | . 007952 | . 007272 | . 002424 |  |
| 61 | .0082017\%0 | . 0075000 | .0025(10) |  |
| 62 | . 00994 | . 009088 | .0030\% |  |
| 63 | .0109356 | . 01 | . 0038999 |  |
| 64 | . 01181173 | . 0108010 | $.003603+$ | U. S. C.S. |
| 65 | . 019424 | . 0113181 | .0037727 |  |
| 66 | . 0131228 | . 012 | . 004 |  |
| 67 | . 013256 | .0121212 | . 0040404 |  |
| 68 | . 01574740 | . 014400 | . 0048000 | U. S. C. S. |
| 69 | . 01640354 | . 015000 | . 005000 | U. S. C. S. |
| 70 | .0218712 | . 02 | .007999 |  |
| 71 | . 02294414 | .0281250 | . 0093750 |  |
| 72 | . 03149480 | . 0288800 | . 0096000 |  |
| 73 | . 03280708 | . 030000 | . 010000 | " " |
| 74 | .036452:31 | . $03333+$ | . $0111111+$ |  |
| 75 | . 04100885 | .035500 | . 012500 |  |
| 76 | . 04686726 | . 0428547 | . 0142849 |  |
| 77 | .04989670 | . 0456273 | . 0152091 |  |
| 78 | .05467847 | . 050000 | . $016066+$ |  |
| 79 | .06561416 | . 060000 | . 020000 |  |
| 80 | . 07873700 | . 072000 | . 024000 | U. S. C S. |
| 81 | .082017\%0 | . 075000 | . 025000 |  |
| 82 | . 10935504 | . 100000 | .033333 + |  |
| 83 | . 13122883 | . 12000 | . 040000 |  |
| 84 | . 164033541 | . 150000 | . 0550000 |  |
| 85 | . 218712 | . 2 | . 079999 |  |
| 86 | - .32807083 | . 30000 | . 100000 | U. S. C. S. |
| 87 | . $6561416+$ | . 600000 | . 200000 |  |
| 88 | 1. | . 914392 | . 304464 |  |
| 89 | 1.093569 | 1. | . 333333 |  |
| 90 | 3.2807083 | 3.00000 | 1.0000 |  |
| 91 | 39.36850 | 36.0000 | 12.0000 |  |
| 92 | $52.43103+$ | 48.00000 | 16.0000 |  |
| 93 | 78.737000 | 72.0000 | 24.0000 |  |

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